Tech Session: *HVE* Accelerometer Output

This Tech Session explains how HVE's **Accelerometer Output** computes the **position, velocity, and acceleration** of a point rigidly attached to the vehicle body such as an accelerometer or any sensor mounted away from the vehicle's center of gravity (CG). By combining translational and rotational motion, it determines the motion an accelerometer experiences anywhere on the vehicle at any instant in the simulation.

Coordinate Systems and Transformations

Two coordinate frames are used:

- **Vehicle-Fixed Coordinates (SAE J670)**: +X forward (longitudinal), +Y right (lateral), +Z down (vertical).
- **Earth-Fixed Coordinates** — an inertial "world" frame.

Position of the Accelerometer

A rotation (attitude) matrix converts vectors from vehicle to Earth coordinates.

The accelerometer's position in the Earth (global) frame is computed as:

$$\mathbf{s} = \mathbf{r}_{CG} + A \cdot \mathbf{r}_{sensor}$$

Where:

- $\mathbf{r}_{CG}$= position of vehicle CG in Earth frame (EarthCG),
- A = direction cosine matrix, which transforms from **vehicle coordinates → Earth coordinates**,
- $\mathbf{r}_{sensor}$= sensor position in vehicle coordinates (VehCoord).

Using **SAE J670 axes** (+X forward, +Y right, +Z down) and the standard **yaw–pitch–roll** (ψ about +Z, θ about +Y, ϕ about +X), the direction cosine matrix for **vehicle → Earth** is:

$$A(\psi, \theta, \phi) = R_z(\psi)\, R_y(\theta)\, R_x(\phi)$$
$$= \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Where:

- ψ= yaw (about +Z),
- θ= pitch (about +Y),
- ϕ= roll (about +X).

Velocity at the Accelerometer

For a rigid body, the velocity at any point equals the CG velocity plus the contribution from rotation.

The local velocity of any point on a rigid body is given by:

$$\mathbf{v}_{sensor} = \mathbf{v}_{CG} + \boldsymbol{\omega} \times \mathbf{r}_{sensor}$$

Here:

- $\mathbf{v}_{CG} = [u, v, w]$ is the CG velocity components (in vehicle axes),
- $\boldsymbol{\omega} = [p, q, r]$ is the angular velocity vector (roll, pitch, yaw rates),
- $\mathbf{r}_{sensor}$ is again the position of the accelerometer relative to the CG.

The cross-product term $\boldsymbol{\omega} \times \mathbf{r}_{sensor}$ adds the **extra velocity** caused by rotation — for example, a sensor further from the center moves faster when the vehicle is turning or rolling.

To express the velocity of the accelerometer in the Earth frame, use the same rotation matrix:

$$\mathbf{V}_{earth} = A \cdot \mathbf{v}_{sensor}$$

This rotates the velocity vector from the vehicle's local coordinate frame into Earth coordinates.

Acceleration at the Accelerometer

The linear acceleration at any point on a rigid body is given by:

$$\mathbf{a}_{sensor} = \mathbf{a}_{CG} + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{sensor} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{sensor})$$

Where:

- $\mathbf{a}_{CG} = [a_{long}, a_{lat}, a_{vert}]$ is the acceleration of the vehicle's CG in vehicle axes,
- $\dot{\boldsymbol{\omega}} = [\dot{p}, \dot{q}, \dot{r}]$ is the angular acceleration vector,
- The double cross product adds **centripetal** and **Coriolis**-type effects due to rotation.

Expanded component-wise, the equations become:

$$\begin{aligned} a_x &= a_{long} - x(q^2 + r^2) + y(pq - \dot{r}) + z(pr + \dot{q}) \\ a_y &= a_{lat} + x(pq + \dot{r}) - y(p^2 + r^2) + z(qr - \dot{p}) \\ a_z &= a_{vert} + x(pr - \dot{q}) + y(qr + \dot{p}) - z(p^2 + q^2) \end{aligned}$$

Each term corresponds to:

- $a_{long}, a_{lat}, a_{vert}$: translational acceleration at the CG,
- $-x(q^2 + r^2)$etc.: centripetal terms (due to steady rotation),
- $y(pq - \dot{r})$, etc.: tangential terms (due to angular acceleration).

Outputs

The function provides:

- $\mathbf{s}$ → sensor position in Earth coordinates,
- $\mathbf{v}_{sensor}$→ velocity in vehicle coordinates (plus its magnitude),
- $\mathbf{V}_{earth}$ → velocity in Earth coordinates,
- **a** → acceleration in vehicle coordinates (plus its magnitude).

These can then be used to:

- Simulate accelerometer readings,
- Compare against measured data,
- Or visualize sensor motion in a simulation.

Summary

Quantity	Equation	Meaning
Position	$\mathbf{s} = \mathbf{r}_{CG} + A\mathbf{r}_{sensor}$	World position of the accelerometer
Velocity (vehicle frame)	$\mathbf{v}_{sensor} = \mathbf{v}_{CG} + \boldsymbol{\omega} \times \mathbf{r}_{sensor}$	Motion due to both translation and rotation
Velocity (earth frame)	$\mathbf{V}_{earth} = A\mathbf{v}_{sensor}$	Velocity in world axes
Acceleration	$\mathbf{a}_{sensor} = \mathbf{a}_{CG} + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{sensor} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{sensor})$	Total kinematic acceleration at the sensor

Worked Example

This example demonstrates how **HVE's Accelerometer Output** computes the vehicle-fixed velocity and acceleration at a point rigidly attached to the vehicle body, following **SAE J670 coordinate conventions** (+X forward, +Y right, +Z down). For this case, the vehicle attitude angles are zero (roll = pitch = yaw = 0), so the direction-cosine matrix is the identity: $A = I_{3\times3}$. As a result, vehicle and Earth axes are aligned and no rotation is applied when mapping vectors to Earth coordinates.

The scenario represents a **braking maneuver with a right-hand yaw rotation**, where an accelerometer is mounted forward, left, and above the vehicle's center of gravity (CG). The inputs describe the motion of the CG and the angular motion of the vehicle body at a specific instant in time.

- **Accelerometer location (r_{sensor})** — The sensor is located **36 in forward**, **24 in left**, and **18 in above** the CG. Its offset determines how much rotational motion contributes to its local acceleration.
- **CG velocity (v_{CG})** — The vehicle is moving **forward at 45 mph (66 ft/s)**. This is the translational speed of the CG along the vehicle's X-axis.
- **CG acceleration (a_{CG})** — The vehicle is **braking at −0.4 g (−12.87 ft/s^2)** longitudinally, with no lateral or vertical components.

- **Yaw rate (r)** — The vehicle is rotating **to the right at 30°/s (0.5236 rad/s)** about its Z-axis. This represents the steady yaw motion.
- **Yaw acceleration ($\dot{r}$)** — The yaw rate is **increasing at 5.7°/s^2 (0.10 rad/s^2)**, introducing a tangential (angular acceleration) component.
- **Roll and pitch rates (p, q)** — Both are zero, indicating the vehicle remains level.

These inputs together define the combined **translational and rotational state** of the vehicle at the CG. Using these quantities, HVE's Accelerometer Output computes the velocity and acceleration at the offset sensor location, capturing both **tangential** (due to changing yaw rate) and **centripetal** (due to steady yaw rotation) effects.

Input Conditions

Parameter	Symbol	Value
Accelerometer location (vehicle coords)	$\mathbf{r}_{sensor}$	(+36, −24, +18) in = (+3.0, −2.0, +1.5) ft
CG velocity	$\mathbf{v}_{CG}$	(+45, 0, 0) mph = (+66, 0, 0) ft/s
CG acceleration	$\mathbf{a}_{CG}$	(−0.40, 0, 0) g = (−12.87, 0, 0) ft/s^2
Rotation rate	$\boldsymbol{\omega}$	(0, 0, +30)°/s = (0, 0, +0.5236) rad/s
Rotation acceleration	$\dot{\boldsymbol{\omega}}$	(0, 0, +5.7)°/s^2 = (0, 0, +0.10) rad/s^2

Step 1 — Velocity at the Accelerometer

$$\mathbf{v}_{sensor} = \mathbf{v}_{CG} + \boldsymbol{\omega} \times \mathbf{r}_{sensor}$$

$\boldsymbol{\omega} \times \mathbf{r}_{sensor}$= (+1.05, +1.57, 0) ft/s

$\mathbf{v}_{sensor}$= (+67.05, +1.57, 0) ft/s

$|\mathbf{v}_{sensor}|$ = 67.1 ft/s = 45.8 mph

With the accelerometer left of the CG (negative Y) and a right-hand yaw, the local forward speed is slightly higher than the CG, with a small rightward component from rotation.

Step 2 — Acceleration at the Accelerometer

$$\mathbf{a}_{sensor} = \mathbf{a}_{CG} + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{sensor} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{sensor})$$

Tangential: $\dot{\boldsymbol{\omega}} \times \mathbf{r}_{sensor}$ = (+0.20, +0.30, 0) ft/s^2

Centripetal: $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{sensor})$ = (−0.82, +0.55, 0) ft/s^2

Total:

$\mathbf{a}_{sensor}$= (−12.87, 0, 0) + (+0.20, +0.30, 0) + (−0.82, +0.55, 0)

$\mathbf{a}_{sensor}$ = (−13.49, +0.85, 0) ft/s^2

$\mathbf{a}_{sensor}$/g = (−0.42, +0.026, 0) g

Interpretation

During a **0.4 g braking maneuver** with a **rightward yaw rate of 30°/s** increasing at **5.7°/s^2**, a Accelerometer mounted **36 in forward, 24 in left, and 18 in above** the CG experiences:

- **0.42 g longitudinal deceleration** (slightly greater than the CG)
- **~0.03 g lateral (+Y)** toward the **right-hand turn center**
- **No vertical acceleration** (vehicle level)

Rotational effects alter accelerations measured away from the CG—these are the kinematic components computed by HVE's **Accelerometer Output**.

Key Takeaways

HVE's Accelerometer Output:

- Computes the motion at any rigidly mounted point on the vehicle.
- Includes translational, tangential (angular-acceleration), and centripetal (steady-rotation) effects.
- Uses consistent SAE conventions; results align with HVE's vehicle-fixed outputs.
- Ideal for virtual IMU placement studies and validation against test data.