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Validation of Accident Reconstruction Formulas Using HVE

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ABSTRACT

Starting with the classic speed-to-stop formula, two formulas used in accident reconstruction are examined using HVE to validate them and their domain of applicability. These include formulas for spinning vehicles and sliding vehicles. A discussion of each formula is presented and along with the results of the relevant HVE validation simulations.

OVERVIEW

Accident reconstruction analysis often uses basic formulas to calculate various quantities of interest, particularly the initial speeds of vehicles. The derivations of these formulas proceed from simplified models of vehicle motion that ignore various complicating factors. It is of interest to examine the accuracy of these formulas and the limits of their use but it is impractical to conduct real experiments with physical vehicles. With the introduction of the HVE SIMON physics module, we can now conduct virtual experiments to determine the proper domain of application for these formulas. In this paper, we study two widely used formulas using SIMON and EDSMAC4 simulation runs and present some observations and conclusions.

SKID TO STOP

The "skid to stop" formula is often used to compute a likely initial speed for a vehicle, given the distance the vehicle slid to a stop on a roadway. It is a consequence of the work-energy theorem of physics, which states that the energy lost by a sliding object must equal the work done. Using the formula for the kinetic energy of a vehicle and setting it equal to the work done against the force of friction yields the formula:

$$v = \sqrt{2 \cdot \mu \cdot g \cdot S}$$

In this formula, v is the speed in feet/second, μ is the coefficient of sliding friction, g is the acceleration of gravity (approximately 32.2 feet/second/second) and S is the distance slid in feet.

This formula can be applied when the model of the vehicle motion is that of a "hockey puck". The derivation of the formula ignores any jounce and rebound, any "weight-shifting", and any other complication of vehicle construction and motion. The question then becomes how accurate is this formula? Can it still be used productively in the analysis of vehicle motion?

To investigate this question, we ran several simulations using both the SIMON physics module and the EDSMAC4 physics module. We used an unmodified 2003 Ford Crown Victoria and an unmodified 1979 Ford Crown Victoria from the standard HVE vehicle database. An interesting aspect of these vehicles is that the 2003 Ford uses an anti-lock braking (ABS) model. For this vehicle, we derived an average coefficient of friction of 0.905, which was then used to compute an initial speed from the skid to stop formula. The various friction properties of the vehicle tires are presented in Table 1 and the results simulations are presented in Table 2. It should be noted that the EDSMAC4 physics module only uses the coefficient of sliding friction at a specific load. In contrast, the SIMON physics module provides a table of coefficient of friction values based on load and speed (up to 9 values for a combination of 3 loads and 3 speeds).

Table 1

Vehicles	μ_{peak}	$\mu_{sliding}$	S_{peak}	ABS
2003 Ford	1.100	0.850	0.160	Yes
1979 Ford	0.843	0.600	0.102	No

For Table 2, four simulation sets (SIMON/EDSMAC4) were run at an initial speed of 15, 30, 45, and 60 mph with the initial speed as the independent variable. The integration time step for the runs was 0.001 seconds and the output time interval was 0.05 seconds. After the runs were completed, the distance covered during very hard braking was computed. In addition, the distance for any displayed skid marks was measured. The goal of the analysis was to determine how accurate a prediction of the initial speed could be if an investigator knew (via some yet to be described technique) the distance covered by braking and in addition, how accurate a prediction of the initial speed could be if just the length of the skid marks was known.

All vehicle braking was initiated 1.0 seconds into the simulation run and reached full braking at 1.1 seconds into the run. This generally means that for the SIMON runs, the vehicles will have slowed slightly from their initial speeds. The actual initial speed when braking was initiated is displayed to the right of the "/" in the predicted speed row. Skid marks were not left for some runs and for those runs, the corresponding table cell values were left blank.

Examining the results of the SIMON runs, we see that even with the complications modeled by the SIMON physics module, the error is less than one percent. We also see that for the 1979 Ford, which does not have ABS activated, the results are generally in good agreement with the EDSMAC4 results.

We derived an effective coefficient of friction for the 2003 Ford by computing the effective coefficients of friction at the four analyzed speeds and then averaging them. For example, the 15 mph run had an effective coefficient of friction of 0.906 and the 60 mph run had an effective coefficient of friction of 0.901. Adding the four effective coefficients of frictions and then averaging them resulted in an average coefficent of friction of 0.905.

Table 2

Distance/Speed	2003 SIMON	2003 EDSMAC4	1979 SIMON	1979 EDSMAC4
15 MPH Distance	7	9	11	13
Predicted Speed	13.77/13.78	15.13	14.06/13.93	15.28
% Error	0.07	0.87	0.93	1.87
15 MPH Skid		8		12
Predicted Speed		14.27		14.68
% Error		4.87		2.13
30 MPH Distance	28	35	47	50
Predicted Speed	27.54/27.74	29.84	29.06/29.09	29.97
% Error	0.72	0.53	0.10	0.10
30 MPH Skid		32	35	47
Predicted Speed		28.54	25.07/29.09	29.06
% Error		4.87	13.82	3.13
45 MPH Distance	71	80	109	113
Predicted Speed	43.86/43.60	45.12	44.25/44.19	45.05
% Error	0.60	0.27	0.14	0.11
45 MPH Skid		77	96	110
Predicted Speed		44.27	41.53/44.19	44.45
% Error		1.62	6.02	1.22
60 MPH Distance	128	141	195	200
Predicted Speed	58.89/58.77	59.90	59.18/58.91	59.94
% Error	0.20	0.17	0.46	0.10
60 MPH Skid		138	178	197
Predicted Speed		59.25	56.45/58.91	59.49
% Error		1.25	4.16	0.85

Consequently, the results for this vehicle can be interpreted as demonstrating that the variance about the average is small. The central issue then, for using the skid to stop formula in ABS situations, is to determine the effective coefficient of friction from ABS and tire parameters. We do not attempt to derive such an effective coefficient in this paper but note that the value of approximately 0.9 is proportionally approximately 20% of the peak value of the coefficient of friction plus approximately 80% of the sliding value of the coefficient of friction as given in Table 1.

ROTATING VEHICLES

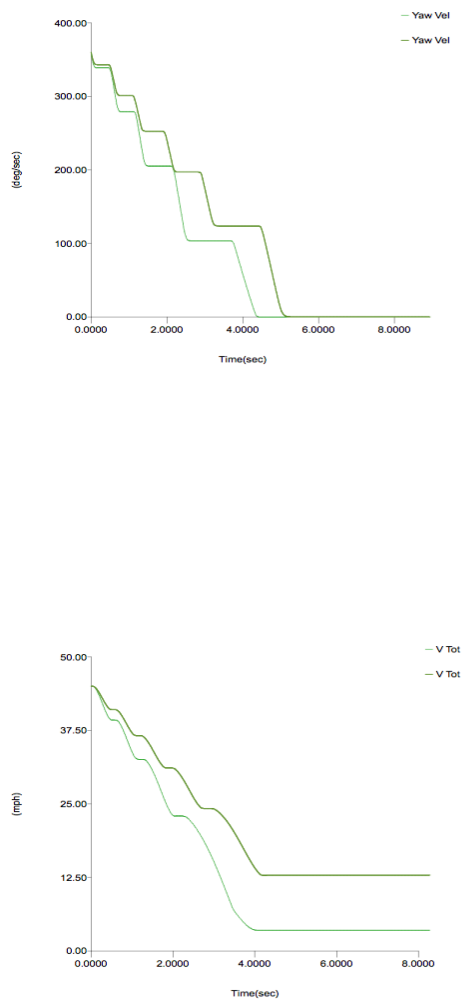
The "skid to stop" formula cannot be used unmodified when the vehicle is rotating. However, a new formula can be derived based on the "bicycle model" of vehicle motion. In this model, the vehicle is reduced to two wheels, free to rotate, that are aligned and connected by rigid strut. For our purposes, the wheels are assumed to slide with friction for any slip angle of the wheel and slide without friction when the wheels are aligned perfectly with the velocity vector of the vehicle. There are three qualitative dynamical regimes for the vehicle.

This model implies that when the vehicle is at first, spinning very fast, it will slide without friction down the roadway. However, the friction of the road will exert a torque on the vehicle, slowing its rotation. As the angular velocity of the vehicle decreases, it will reach a point where the tangent speed of a wheel will become less than the center of mass speed of the vehicle. At this point, the dynamics of the vehicle will shift such that the vehicle will no longer experience a torque but will now slide with friction and lose speed.

As the vehicle slows down, there will come a point when torque will be reintroduced when the tangent speed of the wheels match the center of mass speed. At some point the angular velocity will disappear and the vehicle will enter the third dynamical regime where it simply rolls without friction at some final speed and in some final direction.

To illustrate this we present Figures 1 and 2. Figure 1 displays the angular velocities and total speed of the Ford Crown Victorias as computed by the EDSMAC4 physics module.

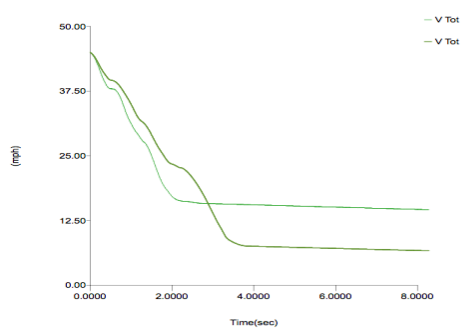
Figure 1 - EDSMAC4



In the above graphs, the darker line represents the results for the 1979 Ford, while the lighter colored line represents the results for the 2003 Ford. We note that we see the expected behavior.

There is no torque exerted on the vehicles most of the time. As the slip angle approaches zero, a torque is introduced for a short period of time and the angular velocity drops. As one can see in the graph of the total speed for the vehicles on the right, during this period there is no frictional force exerted on the vehicles. After this short time interval passes, the slip angle is large enough for the condition of zero torque to start again.

Figure 2 - SIMON



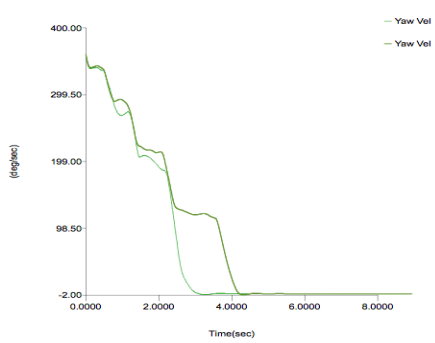


Figure 2 in turn, displays the angular velocity of the Ford Crown Victorias as computed by the SIMON physics module.

We see that the SIMON physics module introduces torques at each moment in time, but there are periods where the torque is small. We also note that for the 2003 Ford, there is no "fourth" plateau of small torque; rather, after the third plateau, the torque drops precipitously to zero. This leads to significant qualitative differences in vehicle motion between EDSMAC4 and SIMON.

In this second dynamical regime of zero torque, the vehicle experiences a frictional force oriented perpendicularly to the wheels and as the vehicle rotates, so does this frictional force. As the vehicle slides down the road, the average component of this force along the direction of the center of mass velocity ("down the road") is a quantity of interest. This average component can be used to derive an analog of the classic skid to stop formula.

The complete derivation of the new formula is presented in Appendix A and yields a form that is still essentially the same as the formula for skidding

$$v = \sqrt{2 \cdot \mu_{eff} \cdot g \cdot S + v_F^{\,2}}$$

In the equation above, v_F is the final speed when the angular velocity goes to 0. However, the coefficient of friction is replaced by an effective coefficient of friction given by

$$\mu_{eff} = \frac{2}{\pi} \cdot \mu \cdot \sqrt{1 - \left(\frac{v_T}{v_{CM}}\right)^2}$$

In this expression for the effective coefficient of friction, v_T is the tangential speed of the wheels in feet/second (approximately given by the angular velocity times half the wheel base) and v_{CM} is the speed of the center of mass. Notice that the formula is speed dependent and reduces to zero when the tangential speed is equal to the center of mass speed. That is the approximate demarcation between the two dynamical regimes. Generally we assume that the tangential speed is slow compared to the center of mass speed and so use the approximate expression:

$$\mu_{eff} = \frac{2}{\pi} \cdot \mu$$

Again we are interested in how accurate is this formula and does it hold up when the complications of actual vehicle motion are included.

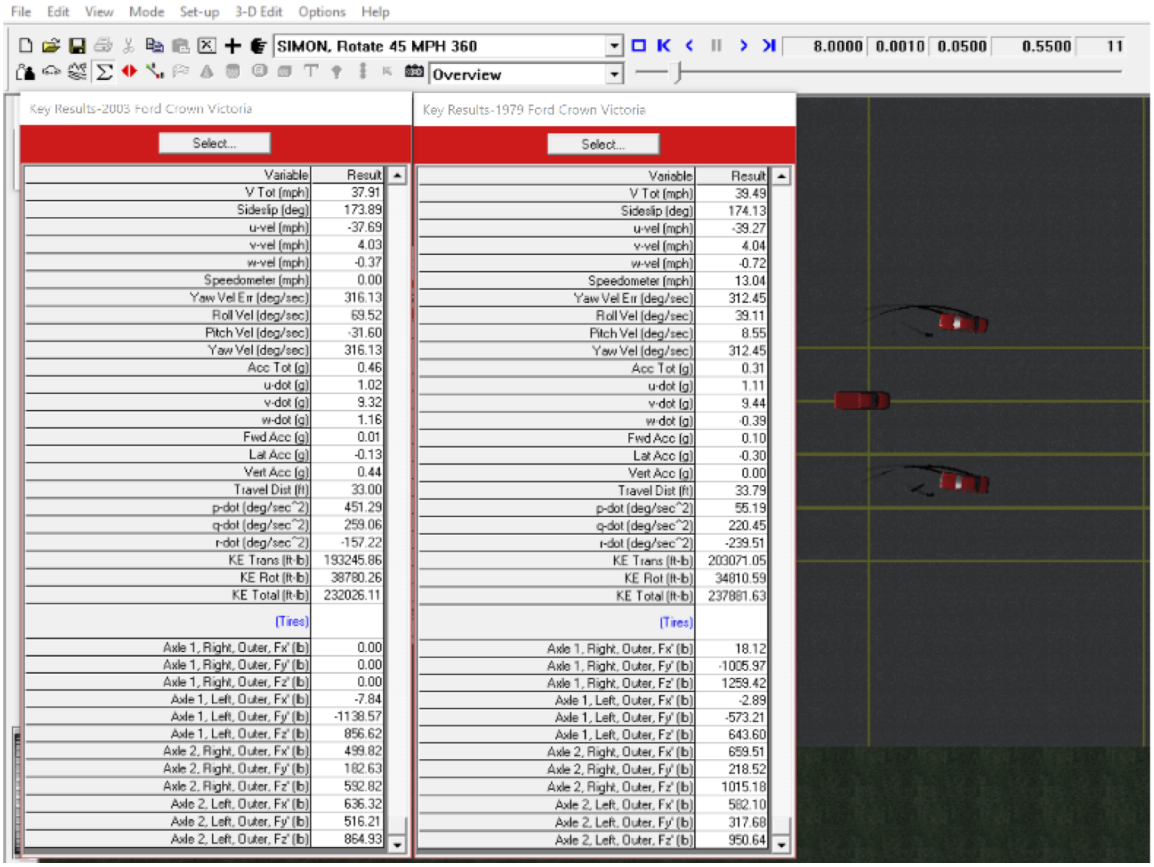
As we did before, we ran several simulations using both the SIMON physics module and the EDSMAC4 physics module. The results are presented in Table 3.

For Table 3, four simulations sets (SIMON/EDSMAC4) were run at an initial speed of 45 mph and initial angular velocities of 360, 270, 180, and 90 degrees per second with the initial angular velocity as the independent variable. The integration time step for the runs was 0.001 seconds and the output time interval was 0.05 seconds.

After the runs were completed, the distance travelled by the vehicles was measured. Because the formula is only valid when the vehicles are rotating, this distance was the distance travelled up to the point of essentially zero angular velocity. The goal of the analysis was to determine how accurate a prediction of the initial speed could be if an investigator knew the final speed at the point of zero angular velocity and the distance travelled while rotating.

For Table 3, the times are in seconds, the speeds are in mph, the distances are in feet, and the azimuths are in degrees.

We can see from the results for the simulation runs using the SIMON physics module, the use of the effective friction formula works well for the 2003 Crown Victoria but leads to an under prediction for the speed of the 1979 Crown Victoria. A qualitative difference can be seen between the 2003 Ford and the 1979 Ford. The final speeds for the 2003 Ford are significant in value and are larger than 15 mph. On the other hand, the final speeds for the 1979 Ford are generally smaller and except for one outlier, are less than 8 mph. This means that the 1979 Ford had a larger change in speed over the sliding interval than the 2003 Ford. It appears that beyond the speed dependency in the original formula for the effective coefficient of friction, there is another dependency that must be accounted for that would increase the effective coefficient of friction. The source of this additional friction can be seen in this screenshot of the 360 º / second SIMON run at T - 0.55 seconds.



Variable	Result
V Tot (mph)	37.91
Sideslip (deg)	173.89
u-vel (mph)	-37.69
v-vel (mph)	4.03
w-vel (mph)	-0.37
Speedometer (mph)	0.00
Yaw Vel Err (deg/sec)	316.13
Roll Vel (deg/sec)	69.52
Pitch Vel (deg/sec)	-31.60
Yaw Vel (deg/sec)	316.13
Acc Tot (g)	0.46
u-dot (g)	1.02
v-dot (g)	9.32
w-dot (g)	1.16
Fwd Acc (g)	0.01
Lat Acc (g)	-0.13
Vert Acc (g)	0.44
Travel Dist (ft)	33.00
p-dot (deg/sec^2)	451.29
q-dot (deg/sec^2)	259.06
r-dot (deg/sec^2)	-157.22
KE Trans (ft-lb)	193245.86
KE Rot (ft-lb)	38780.26
KE Total (ft-lb)	232026.11
(Tires)	
Axle 1, Right, Outer, Fx' (lb)	0.00
Axle 1, Right, Outer, Fy' (lb)	0.00
Axle 1, Right, Outer, Fz' (lb)	0.00
Axle 1, Left, Outer, Fx' (lb)	-7.84
Axle 1, Left, Outer, Fy' (lb)	-1138.57
Axle 1, Left, Outer, Fz' (lb)	856.62
Axle 2, Right, Outer, Fx' (lb)	499.82
Axle 2, Right, Outer, Fy' (lb)	182.63
Axle 2, Right, Outer, Fz' (lb)	592.82
Axle 2, Left, Outer, Fx' (lb)	636.32
Axle 2, Left, Outer, Fy' (lb)	516.21
Axle 2, Left, Outer, Fz' (lb)	864.93

Variable	Result
V Tot (mph)	39.49
Sideslip (deg)	174.13
u-vel (mph)	-39.27
v-vel (mph)	4.04
w-vel (mph)	-0.72
Speedometer (mph)	13.04
Yaw Vel Err (deg/sec)	312.45
Roll Vel (deg/sec)	39.11
Pitch Vel (deg/sec)	8.55
Yaw Vel (deg/sec)	312.45
Acc Tot (g)	0.31
u-dot (g)	1.11
v-dot (g)	9.44
w-dot (g)	-0.39
Fwd Acc (g)	0.10
Lat Acc (g)	-0.30
Vert Acc (g)	0.00
Travel Dist (ft)	33.79
p-dot (deg/sec^2)	55.19
q-dot (deg/sec^2)	220.45
r-dot (deg/sec^2)	-239.51
KE Trans (ft-lb)	203071.05
KE Rot (ft-lb)	34810.59
KE Total (ft-lb)	237881.63
(Tires)	
Axle 1, Right, Outer, Fx' (lb)	18.12
Axle 1, Right, Outer, Fy' (lb)	-1005.97
Axle 1, Right, Outer, Fz' (lb)	1259.42
Axle 1, Left, Outer, Fx' (lb)	-2.89
Axle 1, Left, Outer, Fy' (lb)	-573.21
Axle 1, Left, Outer, Fz' (lb)	643.60
Axle 2, Right, Outer, Fx' (lb)	659.51
Axle 2, Right, Outer, Fy' (lb)	218.52
Axle 2, Right, Outer, Fz' (lb)	1015.18
Axle 2, Left, Outer, Fx' (lb)	582.10
Axle 2, Left, Outer, Fy' (lb)	317.68
Axle 2, Left, Outer, Fz' (lb)	950.64

Table 3

Results	2003 SIMON	2003 EDSMAC4	1979 SIMON	1979 EDSMAC4
360 º / second				
Time Interval	2.75	3.95	3.75	4.60
Final Speed	15.77	3.54	7.53	12.90
Distance	107.17	137.15	135.97	186.38
Final Azimuth	570.34	705.37	722.28	907.59
Predicted Speed	44.55	47.27	40.14	47.93
270 º / second				
Time Interval	2.15	3.85	3.65	5.10
Final Speed	19.75	3.96	5.51	5.89
Distance	96.79	133.19	130.11	185.40
Final Azimuth	378.54	535.25	537.86	712.70
Predicted Speed	44.25	46.62	38.96	46.42
180 º / second				
Time Interval	1.50	3.35	3.15	3.55
Final Speed	27.27	7.58	11.20	22.00
Distance	77.38	127.08	128.80	162.78
Final Azimuth	187.91	353.26	362.21	391.30
Predicted Speed	44.69	46.00	39.98	48.43
90 º / second				
Time Interval	1.75	2.90	3.55	2.75
Final Speed	32.18	12.40	3.78	24.22
Distance	96.74	122.91	126.00	136.92
Final Azimuth	59.97	178.80	134.50	184.76
Predicted Speed	51.02	46.31	38.15	46.39

It is easily seen that at this moment in time, there are significant non-zero values for F_x, the forces on the tires aligned with the wheel. These forces do not exist in our bicycle model and do not exist in EDSMAC4. In EDSMAC4, the forward forces F_x are always zero for all tires. We see that in SIMON, these forces are sizeable. For example, the forward force for the left tire on the rear axle is 659 lbs along with a lateral force of 218 lbs. With a normal force of 1015 lbs, the effective coefficient of friction is approximately 0.68, which is larger than the nominal 0.6.

These forces ultimately arise from the fact that the wheel has a non-zero moment of inertia. This means that the wheel will not instantly spin up as longitudinal forces are applied and consequently, there will be a longitudinal force of friction, F_x, on the tire of the wheel. This frictional force will be an important factor in the motion of a vehicle if the time scale of the vehicle's rotation is comparable to the time scale of the wheel's rotation. A basic SIMON run shows that the time for a wheel to spin down from 45 mph is approximately 0.25 seconds under a load of approximately 900 pounds. For a vehicle rotating at 360 degrees per second, the time for a wheel to sweep through a quadrant ($0 < \theta < \pi/2$) is of course, also 0.25 seconds. The fact that these two times are comparable means that the assumption of zero spin inertia for the wheels is unwarranted.

Consequently, SIMON demonstrates that the simple bicycle model does not take into account important additional sources of friction due to wheel moment of inertia. These can act over time to decelerate the vehicle beyond that predicted by our model and so the use of the effective coefficient of friction formula must be tempered by these concerns.

CONCLUSION

After examining the skid to stop formula and the effective coefficient of friction formula, we find that both formulas are accurate with certain qualifications. The skid to stop formula must use an effective coefficient of friction to be accurate in the case of a vehicle using ABS braking. This is not unexpected. This effective coefficient of friction has a higher value than the coefficient of sliding friction but is not as high as the coefficient of peak friction. In the case we examined, the effective coefficient was approximately 6% higher than the value for sliding friction.

Rotating vehicles will decelerate more slowly than the nominal value of sliding friction would predict. The nominal coefficient of friction must be replaced with an effective coefficient of friction that is approximately $2/\pi$ that of the original sliding friction. However our example indicates that the factor must be increased by approximately 25% to 40% when the rotating vehicle slides for a longer time and distance, resulting in a lower final speed. This is due to additional sources of friction, which are not taken into account by the bicycle model of vehicle motion.

We also find that the effective friction formula is more accurate when the initial angular velocity is higher than 90

degrees per second. The vehicle should rotate beyond 90 degrees before having the angular velocity drop to zero. Put simply, the more rotations the better.

APPENDIX A

In this appendix, a derivation of the formula for the effective friction of a rotating vehicle is presented. Using the bicycle model, we first discuss the motion of a rotating vehicle in a general fashion.

Assume a bicycle traveling in a direction parallel to the Y axis with a speed V and rotating about the center of mass with an angular velocity ω. Further assume that the center of mass is at the geometric center of the bicycle so that the two wheels of the bicycle are equidistant from the center of mass with a distance of ℓ feet. We define a tangential speed as $V_T = \omega \cdot \ell$.

Using a coordinate system located at the center of mass, at any one moment, one wheel will be located at a positive Y coordinate and the other will be located at an equal and opposite negative Y coordinate. We will call the positive Y coordinate wheel the "leading" wheel and the negative Y coordinate wheel, the "trailing" wheel.

We describe the angular state of the bicycle by using an angle θ, which is the angle of the rigid strut with respect to the Y axis. This is the longitudinal axis of the bicycle. Because the wheels in the bicycle model are inline with the strut, this angle is also the angle the wheels make with respect to the Y axis.

Examining the situation when the angle θ is between 0 and 90 degrees, we note that the total velocity vector of the leading wheel is the vector sum of the center of mass velocity V and the velocity of the wheel relative to the center of mass, V_T. The frictional force acting on the tire of the wheel, will initially act in a direction opposite to the direction of the total velocity vector. However, because the bicycle model assumes that wheels have no moment of inertia, the leading wheel will instantaneously spin up in such a manner that the frictional force on the wheel will only act perpendicular to the wheel, either to the right or left of the direction defined by the longitudinal axis. Consequently the frictional force acting on the leading wheel, will only act in a transverse direction, either positive or negative.

The sign of the direction of this force is given by the sign of the expression

$$V \cdot \sin(\theta) - V_T$$

Note that for the trailing wheel, the analogous expression adds the two quantities together rather than subtracts them, and consequently is always positive.

If the above expression is positive, then the bicycle is rotating slowly enough that a net drag force acts on the center of mass to slow the sliding vehicle down. In this dynamical regime, the net torque on the vehicle is zero.

If on the other hand, the expression is negative, then the vehicle is rotating fast enough that there is a torque on the vehicle slowing its rotation. In this other dynamical regime, the net force on the vehicle is zero and there is no slowing of the center of mass.

The formula in question is applicable when the vehicle is rotating slowly and

the center of mass is slowing down. For this regime, we wish to average the force along Y axis and divide it by the normal force to calculate an effective coefficient of friction. For an angle θ, the component of the force along the Y axis is $\mu \cdot N \cdot \sin(\theta)$. Consequently we wish to compute

$$\mu_{eff} = \frac{\int_{\theta_C}^{\pi/2} \mu \cdot N \cdot \sin(\theta) d\theta}{N \cdot \int_{0}^{\pi/2} d\theta}$$

where θ_C is the critical angle when the sign of the previously discussed expression changes. That angle is given by

$$\theta_C = \sin^{-1}\left(\frac{V_T}{V}\right)$$

The need for this critical angle in the integral is perhaps a subtle point. Note that for a given angular velocity, no matter how small, there is always a value of θ for which the product of $V \cdot \sin(\theta)$ is less than V_T. Below this value of theta, the previously discussed expression is negative and the vehicle is in the regime of zero force and non-zero torque. Above this value, the vehicle is in the dynamical regime of non-zero force and zero torque. Consequently the integral over the component of the force must be restricted to those values of theta. As the vehicle rotates, it switches between regimes. This "regime change" is clearly illustrated in the graphs of Figure 1.

Evaluation of the integral is straightforward with the previously presented result of

$$\mu_{eff} = \frac{2}{\pi} \cdot \mu \cdot \sqrt{1 - \left(\frac{V_T}{V}\right)^2}$$

It is easily seen that this formula has the correct limit as the tangential speed V_T approaches the center of mass speed V. At that point, there is always a torque and never a force on the center of mass. Conversely, as the tangential speed approaches zero, the effective coefficient of friction approaches the value of $2 \cdot \mu/\pi$ used in our study. Of course, it should be noted that in case, it take an unphysical infinite amount of time to rotate 90 degrees.